First derivative and second derivative test

The First Derivative Test, Second Derivative Test, and Candidates Test are essential tools in AP Calculus AB and BC for analyzing the behavior of functions. These tests help you determine where functions reach local maxima, minima, and absolute extrema—key concepts that appear frequently in both exams. Mastering these techniques allows you to confidently approach problems involving critical points, concavity, and optimization, ensuring a deeper understanding of function analysis and boosting your performance on the AP Calculus exams. By studying “How to Use the First Derivative Test, Second Derivative Test, and Candidates Test” for the AP Calculus AB and BC exams, you should learn to identify and classify local maxima and minima using the First Derivative Test. You should understand how to determine concavity and find points of inflection with the Second Derivative Test. Additionally, you should be able to apply the Candidates Test to locate absolute extrema on a closed interval, enabling you to effectively optimize functions in different scenarios. The First Derivative Test helps determine where a function has local maxima and minima by analyzing the sign changes of the first derivative (f′(x)). Steps to Use the First Derivative Test: Find the First Derivative: Compute f′(x) of the given function f(x). Determine Critical Points: Set f′(x) = 0 and solve for x. Include any points where f′(x) does not exist (but f(x) does). Sign Chart Analysis: Choose test points in the intervals between the critical points. Plug these points into f′(x) to determine if f′(x) is positive or negative in each interval. Interpret Results: Local Maximum: If f′(x) changes from positive to negative at a critical point, f(x) has a local maximum there. Local Minimum: If f′(x) changes from negative to positive at a critical point, f(x) has a local minimum there. No Local Extrema: If f′(x) does not change sign, there is no local extremum at that point. Example: Given f(x) = x3−3x2+4: Compute f′(x) = 3x2−6x. Set 3x2−6x = 0, solve to get critical points: x = 0 and x = 2. Test intervals: For x < 0, f′(x) > 0. For 0 < x < 2, f′(x) < 0. For x > 2, f′(x) > 0. Interpretation: x = 0 is a local maximum. x = 2 is a local minimum. The Second Derivative Test determines the concavity of a function and helps identify local maxima and minima. Steps to Use the Second Derivative Test: Find the Second Derivative: Compute f″(x) from f′(x). Evaluate the Second Derivative at Critical Points: For each critical point x = c found using f′(x) = 0, evaluate f″(c). Interpret Results: Local Minimum: If f″(c) > 0, f(x) is concave up at x = c, indicating a local minimum. Local Maximum: If f″(c) < 0, f(x) is concave down at x = c, indicating a local maximum. Inconclusive: If f″(c) = 0, the test is inconclusive, and you should use the First Derivative Test or another method. Example: Given f(x) = x4−4x3: Compute f′(x) = 4x3−12x2 and f″(x) = 12x2−24x. Critical points from f′(x) = 0: x = 0 and x = 3. Evaluate f″(x): f″(0) = −24, so x = 0 is a local maximum. f″(3) = 36, so x = 3 is a local minimum. The Candidates Test is used to find absolute extrema (global maxima and minima) on a closed interval [a,b]. Steps to Use the Candidates Test: Find the First Derivative and Critical Points: Compute f′(x) and find the critical points within the interval [a,b]. The first step is to differentiate the function f(x) to obtain f′(x). Evaluate the Function at Critical Points: Calculate f(x) at each critical point within [a,b]. For each critical point ccc found in the previous step that lies within the interval, compute the value f(c). Evaluate the Function at Endpoints: Calculate f(a) and f(b). Evaluate the function at the endpoints of the interval. These points are included because the absolute extrema could occur at the boundaries of the interval. Compare Values: The largest value among the critical points and endpoints is the absolute maximum. The smallest value among them is the absolute minimum. Example: Given f(x) = x3−3x2+1 on [−1,2]: Compute f′(x) = 3x2−6x and find critical points x = 0 and x = 2. Evaluate f(x) at: Critical point x = 0: f(0) = 1. Endpoint x = −1: f(−1) = 3. Endpoint x = 2: f(2) = −3. Conclusion: Absolute maximum at x = −1, f(−1) = 3. Absolute minimum at x = 2, f(2) = −3. Consider the function f(x) = x3−6x2+9x+1. First, find the derivative f′(x) = 3x2−12x+9. Setting f′(x) = 0 gives critical points at x = 1 and x = 3. Analyzing the sign changes of f′(x), f′(x) changes from negative to positive at x = 3, indicating a local minimum, and from positive to negative at x = 1, indicating a local maximum. For the function f(x) = 2x4−4x2, first find the first derivative f′(x) = 8x3−8x and set it to zero, giving critical points at x = 0 and x = ±1. Next, compute the second derivative f″(x) = 24x2−8. Evaluating f″(x) at the critical points, f″(1) > 0 and f″(−1) > 0, indicating local minima at x = ±1. f″(0) < 0 indicates a local maximum at x = 0. Analyze the function f(x) = −x3+3x2+12x on the closed interval [−2,3]. First, find the derivative f′(x) = −3x2+6x+12, setting it to zero gives x = −2 and x = 2. Evaluate f(x) at x = −2, x = 2, and the endpoints x = −2 and x = 3. Compare values: f(−2) = −4, f(2) = 32, f(−2) = −40, and f(3) = 27. The absolute minimum is at x = −2, and the absolute maximum is at x = 2. For f(x) = x4−4x3, the first derivative is f′(x) = 4x3−12x2, yielding critical points x = 0 and x = 3. Using the First Derivative Test, f′(x) changes from positive to negative at x = 0, indicating a local maximum, and from negative to positive at x = 3, indicating a local minimum. Applying the Second Derivative Test with f″(x) = 12x2−24x, confirms concavity: f″(0) = −24 (local maximum), and f″(3) = 36 (local minimum). Consider the function f(x) = x3−3x2. The first derivative is f′(x) = 3x2−6x, leading to critical points at x = 0 and x = 2. The second derivative f″(x) = 6x−6 is used to check concavity. At x = 1, f″(1) = 0, suggesting an inflection point where the concavity changes. f″(0) < 0 confirms a local maximum at x = 0, and f″(2) > 0 indicates a local minimum at x = 2. Given the function f(x) = 2x3−9x2+12x+1, which of the following is true about the critical point x = 1 using the First Derivative Test? a) x = 1 is a local maximum.b) x = 1 is a local minimum.c) x = 1 is neither a local maximum nor a local minimum.d) There is no critical point at x = 1. Answer: b) x = 1 is a local minimum. Explanation: To use the First Derivative Test, first find the derivative f′(x) = 6x2−18x+12. Setting f′(x) = 0, we find critical points by solving 6 (x2−3x+2) = 0, giving x = 1 and x = 2. By testing the sign of f′(x) around x = 1, you’ll find that f′(x) changes from negative to positive, indicating that x = 1 is a local minimum. For the function g(x) = x4−4x2, what does the Second Derivative Test reveal about the critical point at x = 0? a) x = 0 is a local maximum.b) x = 0 is a local minimum.c) The test is inconclusive at x = 0.d) There is no critical point at x = 0. Answer: c) The test is inconclusive at x=0. Explanation:First, find g′(x) = 4x3−8x and set g′(x) = 0 to find the critical points: x = 0, x = ±2. Now, find the second derivative: g″(x) = 12x2−8. At x = 0, g″(0) = −8, which suggests concave down, but g′(x) does not change sign near x = 0, making the Second Derivative Test inconclusive. Which of the following correctly applies the Candidates Test to determine the absolute extrema of h(x) = −x3+3x2+1 on the interval [−1,2]? a) The absolute maximum occurs at x = 2, and the absolute minimum occurs at x = −1.b) The absolute maximum occurs at x = −1, and the absolute minimum occurs at x = 2.c) The absolute maximum occurs at x = 1, and the absolute minimum occurs at x = 0.d) The absolute maximum occurs at x = 2, and the absolute minimum occurs at x = 1. Answer: a) The absolute maximum occurs at x=2, and the absolute minimum occurs at x = −1. Explanation: To apply the Candidates Test, find h′(x) = −3x2+6x and set it to zero to find critical points within the interval. This gives x = 0 and x = 2. Evaluate h(x) at x = −1, x = 0, and x = 2: h(−1) = −1, h(0) = 1, h(2) = 5. Thus, the absolute maximum is h(2) = 5 and the absolute minimum is h(−1) = −1. Learning Outcomes Explain the relationship between a function and its first and second derivatives State the second derivative test for local extrema The first derivative test provides an analytical tool for finding local extrema, but the second derivative can also be used to locate extreme values. Using the second derivative can sometimes be a simpler method than using the first derivative. We know that if a continuous function has a local extrema, it must occur at a critical point. However, a function need not have a local extrema at a critical point. Here we examine how the second derivative test can be used to determine whether a function has a local extremum at a critical point. Let [latex]f[/latex] be a twice-differentiable function such that [latex]f^{\prime}(a)=0[/latex] and [latex]f^{\prime \prime}[/latex] is continuous over an open interval [latex]I[/latex] containing [latex]a[/latex]. Suppose [latex]f^{\prime \prime}(a)0[/latex] if [latex]xa[/latex]. Therefore, by the first derivative test, [latex]f[/latex] has a local maximum at [latex]x=a[/latex]. On the other hand, suppose there exists a point [latex]b[/latex] such that [latex]f^{\prime}(b)=0[/latex] but [latex]f^{\prime \prime}(b)>0[/latex]. Since [latex]f^{\prime \prime}[/latex] is continuous over an open interval [latex]I[/latex] containing [latex]b[/latex], then [latex]f^{\prime \prime}(x)>0[/latex] for all [latex]x \in I[/latex] (Figure 9). Then, by Corollary [latex]3, \, f^{\prime}[/latex] is an increasing function over [latex]I[/latex]. Since [latex]f^{\prime}(b)=0[/latex], we conclude that for all [latex]x \in I[/latex], [latex]f^{\prime}(x)0[/latex] if [latex]x>b[/latex]. Therefore, by the first derivative test, [latex]f[/latex] has a local minimum at [latex]x=b[/latex]. Figure 9. Consider a twice-differentiable function [latex]f[/latex] such that [latex]f^{\prime \prime}[/latex] is continuous. Since [latex]f^{\prime}(a)=0[/latex] and [latex]f^{\prime \prime}(a)0[/latex], there is an interval [latex]I[/latex] containing [latex]b[/latex] such that for all [latex]x[/latex] in [latex]I[/latex], [latex]f[/latex] is decreasing if [latex]xb[/latex]. As a result, [latex]f[/latex] has a local minimum at [latex]x=b[/latex]. Suppose [latex]f^{\prime}(c)=0, \, f^{\prime \prime}[/latex] is continuous over an interval containing [latex]c[/latex]. If [latex]f^{\prime \prime}(c)>0[/latex], then [latex]f[/latex] has a local minimum at [latex]c[/latex]. If [latex]f^{\prime \prime}(c) If [latex]f^{\prime \prime}(c)=0[/latex], then the test is inconclusive. Note that for case iii. when [latex]f^{\prime \prime}(c)=0[/latex], then [latex]f[/latex] may have a local maximum, local minimum, or neither at [latex]c[/latex]. For example, the functions [latex]f(x)=x^3[/latex], [latex]f(x)=x^4[/latex], and [latex]f(x)=−x^4[/latex] all have critical points at [latex]x=0[/latex]. In each case, the second derivative is zero at [latex]x=0[/latex]. However, the function [latex]f(x)=x^4[/latex] has a local minimum at [latex]x=0[/latex] whereas the function [latex]f(x)=−x^4[/latex] has a local maximum at [latex]x=0[/latex] and the function [latex]f(x)=x^3[/latex] does not have a local extremum at [latex]x=0[/latex]. Let’s now look at how to use the second derivative test to determine whether [latex]f[/latex] has a local maximum or local minimum at a critical point [latex]c[/latex] where [latex]f^{\prime}(c)=0[/latex]. Use the second derivative to find the location of all local extrema for [latex]f(x)=x^5-5x^3[/latex]. Watch the following video to see the worked solution to Example: Using the Second Derivative Test. Closed Captioning and Transcript Information for Video Consider the function [latex]f(x)=x^3-\left(\frac{3}{2}\right)x^2-18x[/latex]. The points [latex]c=3,-2[/latex] satisfy [latex]f^{\prime}(c)=0[/latex]. Use the second derivative test to determine whether [latex]f[/latex] has a local maximum or local minimum at those points. We have now developed the tools we need to determine where a function is increasing and decreasing, as well as acquired an understanding of the basic shape of the graph. Next we discuss what happens to a function as [latex]x \to \pm \infty[/latex]. At that point, we have enough tools to provide accurate graphs of a large variety of functions. Home>Derivative test – Types, Explanation, and Examples The second derivative test is a systematic method of finding the absolute maximum and absolute minimum value of a real-valued function defined on a closed or bounded interval. The second derivative test can be used in solving optimization problems in physics, economics, engineering. Let us learn more about the second derivative test, steps for the test, uses, and examples on 2nd derivative test. What Is Second Derivative Test? The second derivative test is a systematic method of finding the absolute maximum and absolute minimum value of a real-valued function defined on a closed or bounded interval. Here we consider a function f(x) defined on a closed interval I, and a point x= k belongs to a closed interval (I). Here we consider a function f(x), which is differentiable twice at x = k, then we have the following three conditions. x = k, is a point of local maxima if f'(k) = 0, and f''(k) < 0. The point at x= k is the locl maxima and f(k) is called the local maximum value of f(x). x = k is a point of local minima if f'(k) = 0, and f''(k) >0 . The point at x = k is the local minima and f(k) is called the local minimum value of f(x). The test fails if f'(k) = 0, and f''(k) = 0. And the point x = k is called the point of inflection. Here if the test fails at point x=k, we go back to the first derivative test and once again check if it is the local maxima or the local minima. Steps for Second Derivative Test The following sequence of steps facilitates the second derivative test, to find the local maxima and local minima of the real-valued function. Find the first derivative f'(x) of the function f(x) Equalize the first derivative to zero f'(x) = 0 and find the limiting points \(x_1, x_2\). Find the second derivative of the function f''(x). Substitute the limiting points in the second derivative\(f''(x_1), f''(x_2)\).. If the second derivative is greater than zero\(f''(x_1) > 0\), then the limiting point \((x_1)\) is the local minima. If the second derivative is lesser than zero \(f''(x_2) 0 , and x = 2 is the minima f''(-2) = 6 x (-2) = -12, and f''(-2) < 0, and x = -2 is the maxima. Therefore by using the second derivative test, the local maxima is -2, with a maximum value of f(-2) = 21, and the local minima is 2, with a minimum value of f(2) = -11. Example 2: Find the local maxima and local minima of the function f(x) = x3 - 6x2+9x + 15. using the second derivative test. Solution: The given function is f(x) = x3 - 6x2+9x + 15. f'(x) = 3x2 - 12x + 9 f'(0) = 3(x2 - 4x + 3) x2 - 4x + 3 = 0 or (x - 1)(x - 3)=0. Here x = 1, and x = 3 f''(x) = 6x - 12 f''(1) = 6(1) - 12 = 6 - 12 = -6., f''(1) < 0, and x = 1 is the maxima. f''(3) = 6(3) - 12 = 18 - 12 = 6, f''(3) > 0, and x = 3 is the minima. Therefore by using the second derivative test, the local maxima is 1, with a maximum value of f(1) = 19, and the lock minima is 3, with a minimum value of f(3) = 15 View Answer > go to slidego to slide Breakdown tough concepts through simple visuals. Math will no longer be a tough subject, especially when you understand the concepts through visualizations. Book a Free Trial Class FAQs on Second Derivative Test The second derivative test is a systematic method of finding the local maximum and minimum value of a function defined on a closed interval. Here we consider a function f(x) defined on a closed interval I, and a point x= k in this closed interval. The following are the three outcomes of the second derivative test. x = k, is a point of local maxima if f'(k) = 0, and f''(k) < 0. The point at x= k is the locl maxima and f(k) is called the local maximum value of f(x). x = k is a point of local minima if f'(k) = 0, and f''(k) >0 . The point at x = k is the local minima and f(k) is called the local minimum value of f(x). The test fails if f'(k) = 0, and f''(k) = 0. And the point x = k is called the point of inflection. How Do You Do the Second Derivative Test? The following sequence of steps facilitates the second derivative test, to find the local maxima and local minima of the real-valued function. Find the first derivative f'(x) of the function f(x), and equalize it to zero f'(x) = 0 and find the limiting points \(x_1, x_2\). Find the second derivative of the function f''(x), and substitute the limiting points. If the second derivative is greater than zero\(f''(x_1) > 0\), then the limiting point \((x_1)\) is the local minima, and If the second derivative is lesser than zero \(f''(x_2)